UNIVERSITI SAINS MALAYSIA

Second Semester Examination
Academic Session 2005/2006

April/Mei 2006

IUK 291E – Mathematics II
[Matematik II]

Duration: 3 hours
[Masa: 3 jam]

Please check that this examination paper consists of FIVE pages of printed material before you begin the examination.

Answer FIVE questions. All questions can be answered either in Bahasa Malaysia OR English.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA mukasurat yang bercetak sebelum anda memulakan peperiksaan ini.]

[Jawab LIMA soalan. Semua soalan boleh dijawab dalam Bahasa Malaysia ATAU Bahasa Inggeris.]

1. (a) If f(x,y) = $\sin^{-1} xy$, verify that $f_{xy} = f_{yx}$.

(5 marks)

(b) Find all points on the sphere $x^2 + y^2 + z^2 = 1$ where the tangent plane is parallel to the plane 2x + y - 3z = 2.

(5 marks)

(c) Suppose z = f(x,y) has continuous second-order partial derivatives. If $x = e^r \cos\theta$ and $y = e^r \sin\theta$

show that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = e^{-2r}\left[\frac{\partial^2 z}{\partial r^2} + \frac{\partial^2 z}{\partial \theta^2}\right]$.

(10 marks)

2. (a) Find the volume of the solid region common to the sphere $x^2 + y^2 + z^2 = 4$ and the cylinder r = 2 $\cos\theta$.

(10 marks)

(b) Evaluate the double integral $\int_0^{2\pi} \int_0^a \frac{1}{a+r}\, r\, dr\, d\theta$.

(10 marks)

3. (a) Use variation of parameters to solve the differential equation $y'' - y = 2\cos^2 x$ that satisfies the initial conditions y(0) = 0, $y'(0) = 1$.

(7 marks)

(b) A dairy product produces whole milk and skim milk in quantities x and y liters, respectively. Suppose the price (in ringgit) of whole milk is p(x) = 100 – x and that of skim milk is q(y) = 100 – y, and also assume that C(x, y) = $x^2 + xy + y^2$ is the joint-cost function of the commodities. Maximize the profit P(x, y) = p x + q y – C(x, y).

(7 marks)

(c) Use the Lagrange multipliers to find the minimum distance from the origin to the ellipse $5x^2 - 6xy + 5y^2 = 4$.

(6 marks)

4. (a) Use the binomial series to obtain the power series expansion of $\frac{1}{\sqrt{(1-x^2)}}$.

(10 marks)

(b) Use partial fractions to find the Maclaurin series for the function $f(x) = \frac{2x-3}{x^2-3x+2}$.

(10 marks)

5. (a) Use the Ratio Test to test the series $\sum_{k=1}^{\infty} \frac{k^k}{k!}$ for convergence.

(Hint: $\lim_{k\to\infty}\left(1+\frac{1}{k}\right)^k = e$).

(6 marks)

(b) What is a p-series? Specify the value p and tell whether the p-series $\sum_{k=1}^{\infty} \frac{1}{2k\sqrt{k}}$ converges or diverges.

(6 marks)

(c) A Cobb-Douglas production function is an output function of the form $Q(x,y) = cx^{\alpha} y^{\beta}$ with $\alpha + \beta = 1$. Show that such a function is maximized with respect to the fixed cost $px + qy = k$ when $x = \frac{\alpha k}{p}$ and $y = \frac{\beta k}{q}$.

(8 marks)

1. (a) *Jika* $f(x,y) = \sin^{-1} xy$, *tentusahkan bahawa* $f_{xy} = f_{yx}$.

(5 markah)

(b) *Cari semua titik diatas sfera* $x^2 + y^2 + z^2 = 1$ *dimana satah tangen adalah selari dengan satah* $2x + y - 3z = 2$.

(5 markah)

(c) *Katakan* $z = f(x,y)$ *mempunyai terbitan separa yang selanjar. Jika* $x = e^r \cos\theta$ *dan* $y = e^r \sin\theta$

tunjukkan $\dfrac{\partial^2 z}{\partial x^2} + \dfrac{\partial^2 z}{\partial y^2} = e^{-2r}\left[\dfrac{\partial^2 z}{\partial r^2} + \dfrac{\partial^2 z}{\partial \theta^2}\right]$.

(10 markah)

2. (a) *Cari isipadu pepejal bagi rantau yang sama pada sfera* $x^2 + y^2 + z^2 = 4$ *dan silinder* $r = 2\cos\theta$.

(10 markah)

(b) *Nilaikan kamiran berganda* $\int_0^{2\pi} \int_0^{a} \dfrac{1}{a+r}\, r\, dr\, d\theta$.

(10 markah)

3. (a) *Dengan menggunakan ubahan parameter, selesaikan persamaan pembezaan* $y'' - y = 2\cos^2 x$ *yang memenuhi syarat awal* $y(0) = 0,\ y'(0) = 1$.

(7 markah)

(b) *Satu hasil tenusu mengeluarkan susu pekat dan susu cair dalam kuantiti x dan y liter, masing-masing. Katakan harga (dalam ringgit) bagi susu pekat ialah* $p(x) = 100 - x$ *dan bagi susu cair ialah* $q(y) = 100 - y$, *dan anggapkan juga bahawa* $C(x, y) = x^2 + xy + y^2$ *ialah kos bersama bagi kedua-dua komoditi. Maksimumkan keuntungan* $P(x, y) = p\,x + q\,y - C(x, y)$.

(7 markah)

(c) *Guna pendarab Lagrange untuk mencari jarak minimum dari asalan ke elips* $5x^2 - 6xy + 5y^2 = 4$.

(6 markah)

4. (a) *Guna siri binomial untuk mendapatkan kembangan siri kuasa bagi* $\frac{1}{\sqrt{(1-x^2)}}$.

(10 markah)

(b) *Guna pecahan separa untuk mencari siri Maclaurin bagi fungsi* $f(x) = \frac{2x-3}{x^2-3x+2}$.

(10 markah)

5. (a) *Guna Ujian Nisbah untuk menguji siri* $\sum_{k=1}^{\infty} \frac{k^k}{k!}$ *bagi penumpuan.*

(Petunjuk: $\lim_{k\to\infty}\left(1+\frac{1}{k}\right)^k = e$*).*

(6 markah)

(b) *Terangkan siri -p? Dapatkan nilai p dan nyatakan samada siri-p bagi* $\sum_{k=1}^{\infty} \frac{1}{2k\sqrt{k}}$ *menumpu atau mencapah.*

(6 markah)

(c) *Suatu fungsi pengeluaran Cobb-Douglas ialah suatu fungsi output dalam bentuk* $Q(x,y) = cx^{\alpha}y^{\beta}$ *dengan* $\alpha+\beta=1$. *Tunjukkan fungsi tersebut dimaksimumkan terhadap kos tetap* $px + qy = k$ *bila* $x = \frac{\alpha k}{p}$ *dan* $y = \frac{\beta k}{q}$.

(8 markah)

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